International Journal of Theoretical Physics, Vol. 23, No. 10, 1984

The Hypothesis of the $V - A$ Fermion Current Dominance and Neutral Current Interactions

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Received March 31, 1983

The hypothesis of the $V-A$ fermion current dominance is expressed through an appropriate hypercharge partitioning criterion. The application of this criterion to the standard electroweak model imposes on the Weinberg mixing angle a bound which is in excellent agreement with experiments.

As is well known, in ordinary weak interactions the fermion $V-A$ currents exhibit the total dominance over the $V+A$ currents; i.e., the $V-A$ fermion currents are the only ones coupled to the charged vector boson fields. Hence it stands to reason that the so-called neutral current interactions could also exhibit some kind of fermion $V-A$ current dominance, although it is not expected to be the total dominance as in ordinary weak interactions.

Now if we are to formulate the hypothesis of the fermion $V-A$ current dominance, it will have to have validity beyond the usual neutral current interactions. That is, it should be applicable to all neutral currentlike interactions, especially to the electromagnetic interactions where $V-A$ and $V+A$ fermion currents couple with the same strength to the photon gauge field. (Incidentally, this shows the impossibility of having the same kind of total $V-A$ dominance as in the ordinary weak interactions.)

To arrive at the criterion which will express the hypothesis of the fermion $V-A$ current dominance, let us start with neutral currentlike interaction Lagrangian density

$$\mathscr{L}_{\text{int}} = \lambda j_\mu B^\mu \tag{1}$$

where j^μ is some "neutral current," bilinear in fermion (lepton or quark) fields, B^{μ} is a neutral vector boson field, while λ is some coupling constant. Now, writing down the $U(1)$ generator as

$$Y=\int d^{3}xj^{4}(\mathbf{x},t) \tag{2a}$$

we define the hypercharges $y(\psi_L)$ and $y(\psi_R)$ associated with fermion field ψ as

$$[Y,\psi_{L,R}]=-y(\psi_{L,R})\psi_{L,R} \tag{2b}$$

where $2\psi_{L,R}=(1\pm\gamma_5)\psi$. With $y(\psi_{L,R})$ real, we shall always have

$$\tfrac{1}{2}|y(\psi_L)+y(\psi_R)|+\tfrac{1}{2}|y(\psi_L)-y(\psi_R)|=\begin{cases}|y(\psi_L)| & \text{(3a)}\\ |y(\psi_R)| & \text{(3b)}\end{cases}$$

where (3a) holds if $|y(\psi_L)|\geqslant|y(\psi_R)|$, while (3b) holds if $|y(\psi_L)|\leqslant|y(\psi_R)|$. As the $V-A$ and $V+A$ couplings of ψ to B^{μ} in (1) are proportional to $y(\psi_L)$ and $y(\psi_R)$, respectively, it is clear then that

$$\tfrac{1}{2}|y(\psi_L)+y(\psi_R)|+\tfrac{1}{2}|y(\psi_L)-y(\psi_R)|=|y(\psi_L)| \tag{4}$$

is the desired hypercharge partitioning criterion reflecting generally the dominance of the fermion $V-A$ currents over the $V+A$ currents in (1).

It is now easy to check the correctness of (4) on some simple cases. If the negative helicity neutrinos $[y(\nu)\equiv y(\nu_L),\ y(\nu_R)\equiv 0]$, which are still believed to be the only ones in nature, are present in (1), then (4) [that is, (3a)] is satisfied trivially [while also (3b) is violated trivially]. In electromagnetic interactions where charges satisfy $q_L=q_R$ (we use here q instead of y), relation (4) is also satisfied although (3b) is not violated either.

The unified gauge models which, in addition to the left-handed neutrinos, admit also the right-handed ones $[y(\nu_R)\neq 0]$ need not violate (4). However, in general they may find this hypercharge partitioning criterion to be too stringent. On the other hand, the unified gauge models which admit only the left-handed neutrinos such as the W–S–G (Weinberg, Salam, and Glashow) electroweak theory (Weinberg, 1967; Salam, 1968; Glashow, 1961; Glashow et al., 1970) [the underlying gauge group being $SU(2)_L\times U(1)$] or the electrodual electroweak model (Šoln, 1980a, b) [the underlying gauge group being $SU(2)_L\times U(1)\times U'(1)$] *a priori* have no reason to be inconsistent with the hypothesis of the fermion $V-A$ current dominance (4). In our analysis, however, we shall restrict ourselves only to the W–S–G model (Weinberg, 1967; Salam, 1968; Glashow, 1961; Glashow et al., 1970), while the discussion of the other model (Šoln, 1980a, b) will be done elsewhere.

In the W–S–G electroweak theory the neutral currentlike sector consists of the electromagnetic interactions [which we already verified as satisfying (4)] and the ordinary neutral current interactions whose Lagrangian density is (Weinberg, 1967; Salam, 1967; Glashow, 1961; Glashow et al., 1970) (with Z^{μ} now being the neutral vector boson)

$$\mathscr{L}_{NC} = \lambda j^{\mu}_{NC} Z_{\mu} \tag{5a}$$

Here (for simplicity we restrict ourselves to just four leptons and four quarks)

$$j^{\mu}_{NC} = 2\left[j^{\mu}_{3,L} - \sin^2\theta_W j^{\mu}_{EM} \right] \tag{5b}$$

$$\begin{aligned} j^{\mu}_{EM} = & -\bar{e}\gamma^{\mu}e - \bar{\mu}^{\gamma\mu}\mu + \tfrac{2}{3}\left[\bar{u}\gamma^{\mu}u + \bar{c}^{\gamma\mu}c\right] \\ & -\tfrac{1}{3}\left[\bar{d}\gamma^{\mu}d + \bar{s}\gamma^{\mu}\bar{s}\right] \end{aligned} \tag{5c}$$

$$\lambda = \frac{e}{\sin 2\theta_w} \tag{5d}$$

In relations (5a–d) θ_w is the Weinberg mixing angle and $j^{\mu}_{L,3}$ is the third component of the weak isospin current whose structure is determined by the first and the second generation fermion doublets, respectively (where we ignore the Cabibbo rotation),

$$\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}, \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \tag{6a}$$

$$\begin{pmatrix} \nu_{\mu} \\ \mu_L \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \tag{6b}$$

Now according to relations (2a, b) the hypercharges that correspond to j^{μ}_{Nc} are $(y(\nu_R) \equiv 0)$:

$$\begin{aligned} & y(\nu_e) = 1 \\ & y(e_L) = 2\sin^2\theta_W - 1, \qquad y(e_R) = 2\sin^2\theta_W \\ & y(u_L) = 1 - \tfrac{4}{3}\sin^2\theta_W, \qquad y(u_R) = -\tfrac{4}{3}\sin^2\theta_W \\ & y(d_L) = \tfrac{2}{3}\sin\theta_W - 1, \qquad y(d_R) = \tfrac{2}{3}\sin^2\theta_W \end{aligned} \tag{7}$$

where, understandably, it is sufficient to consider the first generation fermions only.

As far as the neutrino ν_e is concerned, relation (4) does not affect θ_W. However, for other fermions the application of (4)–(7) gives

$$e: \quad 0 \leqslant \sin^2\theta_W \leqslant \tfrac{1}{4} \tag{8a}$$

$$u: \quad 0 \leqslant \sin^2\theta_W \leqslant \tfrac{3}{8} \tag{8b}$$

$$d: \quad 0 \leqslant \sin^2\theta_W \leqslant \tfrac{3}{4} \tag{8c}$$

Hence in order that all fermions (of both generations) exhibit $V-A$ current dominance simultaneously θ_W has to be restricted by $0 \leqslant \sin^2\theta_W \leqslant 0.25$. As experimentally $\sin^2\theta_W$ appears to be between 0.21 and 0.23, bound (8a) is in excellent agreement with experiments.

The calculations of $\sin^2\theta_W$ within the Georgi–Glashow grand unified model with gauge group $SU(5)$ (Georgi and Glashow, 1974; Georgi et al., 1974) will yield the value of 3/8, which interestingly is the upper bound in (8b). However, once the renormalization corrections are taken into account (Kang, 1980) $\sin^2\theta_W$ is found to satisfy bound (8a).

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